MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 29

Total Marks : 33

Max. Time : 33 min.

Topics : Continuity & Derivability, Straight Line, Application of Derivatives, Method of Differentiation

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2	**(3 marks, 3 min.)**	**[6, 6]**
Multiple choice objective (no negative marking) Q.3,4,5	**(5 marks, 4 min.)**	**[15, 12]**
Subjective Questions (no negative marking) Q.6,7,8	**(4 marks, 5 min.)**	**[12, 15]**

1. Let f(x) be defined as follows :

$$f(x) = \begin{cases} (\cos x - \sin x)^{\operatorname{cosec} x} & , \quad -\dfrac{\pi}{2} < x < 0 \\ a & , \quad x = 0 \\ \dfrac{e^{1/x} + e^{2/x} + e^{3/x}}{ae^{2/x} + be^{3/x}} & , \quad 0 < x < \dfrac{\pi}{2} \end{cases}$$

If f(x) is continuous at x = 0, then (a, b) =

(A) $\left(e, \dfrac{1}{e}\right)$ (B) $\left(\dfrac{1}{e}, e\right)$ (C) (e, e) (D) (e^{-1}, e^{-1})

2. If $ax^2 + bx + c = 0$ has imaginary roots and $a - b + c > 0$, then the set of points (x, y) satisfying the equation

$$\left| a\left(x^2 + \frac{y}{a}\right) + (b+1)x + c \right| = |ax^2 + bx + c| + |x + y|$$

consists of the region in the xy-plane which is
(A) on or above the bisector of I and III quadrant (B) on or above the bisector of II and IV quadrant
(C) on or below the bisector of I and III quadrant (D) on or below the bisector of II and IV quadrant

3. Equation of a tangent to the curve $y \cot x = y^3 \tan x$ at the point where the abscissa is $\pi/4$, is:
(A) $4x + 2y = \pi + 2$ (B) $4x - 2y = \pi + 2$
(C) $x = 0$ (D) $y = 0$

4. If the tangent to the curve $2y^3 = ax^2 + x^3$ at the point (a, a) cuts off intercepts α, β on co-ordinate axes, where $\alpha^2 + \beta^2 = 61$, then the value of 'a' is equal to :
(A) 20 (B) 25 (C) 30 (D) – 30

5. The equation of tangents to the curve $y = \cos(x + y)$, $-2\pi \le x \le 2\pi$, that are parallel to the line $x + 2y = 0$ is/are :
(A) $x + 2y = \pi/2$ (B) $x + 2y = -3\pi/2$ (C) $x - 2y = \pi/2$ (D) $x - 2y = -3\pi/2$

6. If $\left(\dfrac{x+b}{2}\right) = a \tan^{-1}(a\,\ell n\, y)$, $a > 0$, then prove that $yy'' - yy'\,\ell ny = (y')^2$

7. Find the equation of the normal to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$ at x = 0

8. If $x = a(t + \sin t)$, $y = a(1 - \cos t)$, then find

(i) $\dfrac{dy}{dx}$ (ii) $\dfrac{d^2y}{dx^2}$ (iii) $\dfrac{d^3y}{dx^3}$

Answers Key

1. (B) **2.** (B) **3.** (A)(B)(D) **4.** (C)(D)

5. (A)(B) **7.** $y + x - 1 = 0$

8. If(i) $\tan \frac{t}{2}$ (ii) $\frac{1}{2a} \sec^4 \left(\frac{t}{2}\right)$

(iii) $\frac{1}{a^2} \sec^6 \left(\frac{t}{2}\right) \tan \left(\frac{t}{2}\right)$